6 Dynamics II: Motion in a Plane

6.1 Kinematics in Two Dimensions

1. Complete the motion diagram for this trajectory, showing velocity and acceleration vectors.

2. A particle moving along a trajectory in the xy-plane has the x-versus-t graph and the y-versus-t graph shown below.

   a. Use the grid below to draw a y-versus-x graph of the trajectory.

   b. Draw the particle's velocity vector at $t = 3.5 \text{ s}$ on your graph.
3. The trajectory of a particle is shown below. The particle's position is indicated with dots at 1 second intervals. The particle moves between each pair of dots at constant speed.

![Graph of x-versus-t and y-versus-t](image)

a. Draw x-versus-t and y-versus-t graphs for the particle.

![Graphs of x-versus-t and y-versus-t](image)

b. Is the particle's speed between \( t = 5 \text{ s} \) and \( t = 6 \text{ s} \) greater than, less than, or equal to its speed between \( t = 1 \text{ s} \) and \( t = 2 \text{ s} \)? Explain.

\[
|v_{1-2}| = \sqrt{0^2 + \left(\frac{5 \text{ m}}{\text{s}}\right)^2} = \frac{5 \text{ m}}{\text{s}}
\]

\[
|v_{5-6}| = \sqrt{\left(\frac{5 \text{ m}}{\text{s}}\right)^2 + \left(\frac{5 \text{ m}}{\text{s}}\right)^2} = \sqrt{50 \frac{\text{m}^2}{\text{s}^2}} = 7.1 \frac{\text{m}}{\text{s}}
\]

The speed between \( t = 5 \text{ s} \) and \( t = 6 \text{ s} \) is greater.

4. The figure shows a ramp and a ball that rolls along the ramp. Draw vector arrows on the figure to show the ball's acceleration at each of the lettered points A to E (or write \( \vec{a} = \vec{0} \), if appropriate).

![Diagram of ball on ramp](image)
6.2 Dynamics in Two Dimensions

5. An ice hockey puck is pushed across frictionless ice in the direction shown. The puck receives a sharp, very short-duration kick toward the right as it crosses line 2. It receives a second kick, of equal strength and duration but toward the left, as it crosses line 3. Sketch the puck’s trajectory from line 1 until it crosses line 4.

6. A rocket motor is taped to an ice hockey puck, oriented so that the thrust is to the left. The puck is given a push across frictionless ice in the direction shown. The rocket will be turned on by remote control as the puck crosses line 2, then turned off as it crosses line 3. Sketch the puck’s trajectory from line 1 until it crosses line 4.
7. An ice hockey puck is sliding from west to east across frictionless ice. When the puck reaches the point marked by the dot, you’re going to give it one sharp blow with a hammer. After hitting it, you want the puck to move from north to south at a speed similar to its initial west-to-east speed. Draw a force vector with its tail on the dot to show the direction in which you will aim your hammer blow. The blow will be at 45° South of West in order to stop the Eastward motion and impart an equal Southward motion.

8. Tarzan swings through the jungle by hanging from a vine.
   a. Draw a motion diagram of Tarzan, as you learned in Chapter 1. Use it to find the direction of Tarzan’s acceleration vector $\vec{a}$:
      i. Immediately after stepping off the branch, and
      ii. At the lowest point in his swing.

b. At the lowest point in the swing, is the tension $T$ in the vine greater than, less than, or equal to Tarzan’s weight? Explain, basing your explanation on Newton’s laws.

For the acceleration to be upwards, the net force must be upwards. The only two forces are the upward tension and the weight. Therefore, the tension must be greater than the weight.
6.3 Projectile Motion

9. A projectile is launched over horizontal ground at an angle between 0° and 90°.
   a. Is there any point on the trajectory where \( \vec{v} \) and \( \vec{a} \) are parallel to each other? If so, where?
      No, unless the launch angle is 90°, \( \vec{a} \) is always vertically downward. \( \vec{v} \) will be parallel to \( \vec{a} \) only if \( \vec{v} \) has no horizontal component, which is only possible if the launch angle is 90°.
   b. Is there any point where \( \vec{v} \) and \( \vec{a} \) are perpendicular to each other? If so, where?
      At the top of the trajectory, \( \vec{v} \) is horizontal, \( \vec{a} \) is vertical.
   c. Which of the following remain constant throughout the entire trajectory: \( r, x, y, v, v_x, v_y, a_x, a_y \)?
      \( v_x, a_x, a_y \) \( (a_x = 0, a_y = -g) \)

10. The figure shows a ball that rolls down a quarter-circle ramp, then off a cliff. Sketch the ball’s trajectory from the instant it is released until it hits the ground.

11. a. A cart that is rolling at constant velocity fires a ball straight up. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in the tube? Explain.
    The ball will land directly in the cart. The ball is already moving horizontally with the cart when launched. Though the force on the ball when launched is vertical, the ball retains the same constant horizontal velocity as the cart and lands in it.
   b. Will your answer change if the cart is accelerating in the forward direction? If so, how?
      If the cart is accelerating, then the ball will land behind the cart. The ball’s horizontal velocity when launched is the same as the cart’s at that moment, but the cart is speeding up and gets ahead of the ball.
12. A rock is thrown from a bridge at an angle 30° below horizontal.
   a. Sketch the rock’s trajectory on the figure.
   b. Immediately after the rock is released, is the magnitude of its acceleration greater than, less than, or equal to \( g \)? Explain.

   \[
   \text{The acceleration of the rock is independent of the initial velocity of the rock.}
   \]

   c. At the instant of impact, is the rock’s speed greater than, less than, or equal to the speed with which it was thrown? Explain.

   \[
   \text{The rock's speed is greater at the instant of impact. The horizontal speed does not change but the vertical speed increases due to gravity.}
   \]

13. Four balls are simultaneously launched with the same speed from the same height \( h \) above the ground. At the same instant, ball 5 is released from rest at the same height. Rank in order, from shortest to longest, the amount of time it takes each of these balls to hit the ground. Ignore air resistance. (Some may be simultaneous.)

   **Order:** 4, 3, 2 = 5, 1

   **Explanation:** Because 1-4 each have the same initial speed, we need only consider the vertical component of their initial velocity. Ball 4 has the greatest initial downward velocity component, Ball 1 the least. Both balls 2 and 5 have no initial velocity in the \( y \)-direction and hit simultaneously.

14. Rank in order, from shortest to longest, the amount of time it takes each of these projectiles to hit the ground. Ignore air resistance. (Some may be simultaneous.)

   **Order:** 1 = 2 = 3 = 4, 5

   **Explanation:** None of the projectiles has any initial velocity in the \( y \)-direction. The time to fall depends only on the height above the ground. Balls 1-4 begin at the same height.
6.4 Relative Motion

15. Anita is running to the right at 5 m/s. Balls 1 and 2 are thrown toward her at 10 m/s by friends standing on the ground. According to Anita, which ball is moving faster? Or are both speeds the same? Explain.

According to Anita, Ball 2 is moving at \(-10 \text{ m/s} - 5 \text{ m/s} = -15 \text{ m/s}\). Ball 1 is moving at \(10 \text{ m/s} - 5 \text{ m/s} = 5 \text{ m/s}\). Ball 2 is faster.

16. Anita is running to the right at 5 m/s. Balls 1 and 2 are thrown toward her by friends standing on the ground. According to Anita, both balls are approaching her at 10 m/s. Which ball was thrown at a faster speed? Or were they thrown with the same speed? Explain.

Ball 1 was thrown at \(15 \text{ m/s}\). Ball 2 was thrown at \(-5 \text{ m/s}\). Thus, according to Anita, ball 2 is moving at \(-5 \text{ m/s} - 5 \text{ m/s} = -10 \text{ m/s}\) and ball 1 is moving at \(15 \text{ m/s} - 5 \text{ m/s} = 10 \text{ m/s}\).

17. Ryan, Samantha, and Tomas are driving their convertibles. At the same instant, they each see a jet plane with an instantaneous velocity of 200 m/s and an acceleration of 5 m/s².

a. Rank in order, from largest to smallest, the jet’s speed \(v_R\), \(v_S\), and \(v_T\) according to Ryan, Samantha, and Tomas. Explain.

The jet’s speed, according to Ryan, is \(200 \text{ m/s} - (-20 \text{ m/s}) = 220 \text{ m/s}\).
The jet’s speed, according to Samantha, is \(200 \text{ m/s} - 20 \text{ m/s} = 180 \text{ m/s}\).
The jet’s speed, according to Tomas, is \(200 \text{ m/s} - 40 \text{ m/s} = 160 \text{ m/s}\).

b. Rank in order, from largest to smallest, the jet’s acceleration \(a_R\), \(a_S\), and \(a_T\) according to Ryan, Samantha, and Tomas. Explain.

\[a_R = a_S = a_T\]

Because Ryan, Samantha, and Tomas are each moving at constant speed, they perceive the same rate of change in speed for the jet. The acceleration of the jet is the same in all inertial reference frames.
18. An electromagnet on the ceiling of an airplane holds a steel ball. When a button is pushed, the magnet releases the ball. The experiment is first done while the plane is parked on the ground, and the point where the ball hits the floor is marked with an X. Then the experiment is repeated while the plane is flying level at a steady 500 mph. Does the ball land slightly in front of the X (toward the nose of the plane), on the X, or slightly behind the X (toward the tail of the plane)? Explain.

The ball still lands on the X. In the second experiment, the ball’s initial velocity relative to the plane is still zero. Although someone at rest on the ground would perceive both the ball and the plane to be moving past at 500 mph, to someone on the plane the ball still falls straight down.

19. Zack is driving past his house. He wants to toss his physics book out the window and have it land in his driveway. If he lets go of the book exactly as he passes the end of the driveway, should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. (Ignore air resistance.)

When Zack throws the book, it also has the forward motion of the car. If he wants the book to follow path 2 into the driveway, he needs to include a backward component to the velocity from his throw that is equal and opposite to the forward velocity of the car.

20. Yvette and Zack are driving down the freeway side by side with their windows rolled down. Zack wants to toss his physics book out the window and have it land in Yvette’s front seat. Should he direct his throw outward and toward the front of the car (throw 1), straight outward (throw 2), or outward and toward the back of the car (throw 3)? Explain. (Ignore air resistance.)

Yvette’s car is not moving relative to Zack’s car. From the perspective of an observer at rest with respect to the freeway, the book already has a forward velocity equal to Yvette’s and, therefore, only needs to be thrown straight outward.